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INVESTIGATION OF THE SOLUTION OF THE PROBLEM FOR A FIRST ORDER, ORDINARY, INTEGRO-DIFFERENTIAL AND BOUNDARY VALUE EQUATION CONTAINING NON-LOCAL AND GLOBAL TERMS IN THE BOUNDARY CONDITION

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In the paper we consider a problem for a first order, general, linear, integro-differential, loaded equation containing linear, general, non-local and global terms in the boundary condition.

By means of the obtained necessary conditions and the boundary condition the loaded terms are removed and the given boundary value problem is reduced to the Fredholm type, the second kind integral equation with weak property.

It is known that the Cauchy and boundary value problems were sufficiently investigated for a first order ordinary differential and integro-differential equation. The equation of the considered boundary value problem contains boundary values of the unknown function, but non-local boundary condition contains global terms.

The solution of the problem is conducted in the following stages.

1) Fundamental solution is constructed for a differential part of the given equation.
2) Necessary condition is found by means of this fundamental solution.
3) The given boundary values of the unknown function are determined from this system (boundary values are expressed by the integral of this unknown function).
4) Finally, having written these found values in the main expression we find Fredholm type integral equation of the second kind for the unknown function.

Problem Statement

Let's consider the following problem:

$$ly \equiv a(x)y'(x) + b(x)y(x) + \int_0^1 \left[K_1(x,\xi)y'(\xi) + K_2(x,\xi)y(\xi)\right] \times$$
$$\times d\xi + b_0(x)y(0) + b_1(x)y(1) = f(x), \quad x \in (0,1). \tag{1}$$

Investigate the solution of equation under the boundary condition

$$\beta_0 y(0)+\beta_1 y(1)+\int_0^1 K(\xi)y(\xi)d\xi=\gamma, \tag{2}$$

here $a(x)\neq 0,\ b(x),\ b_0(x),\ b_1(x)$ and $f(x)$ are the real-valued known continuous functions, $K_1(x,\xi)$ and $K_2(x,\xi)$ are real-valued functions with weak property, $K(x)$ is a real-valued continuous function, $\beta_0,\ \beta_1$ and γ are real constants.

From the adjoint differential solution and its fundamental solution we can easily see that the fundamental solution of the expression

$$l_1 y\equiv a(x)y'(x)+b(x)y(x),$$

adjoint to the principal part of the equation (1)

$$l_1^* z\equiv -[a(x)z(x)]'+b(x)z(x), \tag{3}$$

is

$$Z(x,\eta)=\frac{-\theta(x-\eta)}{a(\eta)}e^{\int_\eta^x \frac{b(\xi)-a'(\xi)}{a(\xi)}d\xi}, \tag{4}$$

here

$$\theta(t)=\begin{cases}1, & t>0,\\ \frac{1}{2}, & t=0,\\ 0, & t<0,\end{cases} \tag{5}$$

is a Heaviside function.

Necessary conditions. The following statement is true.

Theorem 1. *If a* $a(x)\neq 0$ *is a real-valued differentiable function,* $b(x)$, $b_0(x),\ b_1(x)$ and $f(x)$ *are real-valued continuous functions,* $K_1(x,\xi)$ *is a real-valued function, continuous with respect to* x *and continuously differentiable with respect to* ξ *,* $K_2(x,\xi)$ *and* $K(\xi)$ *are real-valued continuous functions,* $\beta_0,\ \beta_1$ and γ *are real constants, then*

$$\int_0^1 f(x)Z(x,\eta)dx-y(1)\Big\{a(1)Z(1,\eta)+\int_0^1 Z(x,\eta)K_1(x,1)dx+$$

$$+\int_0^1 b_1(x)Z(x,\eta)dx\Big\}+y(0)\Big\{a(0)Z(0,\eta)+\int_0^1 Z(x,\eta)K_1(x,0)dx-$$

$$-\int_0^1 b_0(x)Z(x,\eta)dx\ \Big\}+\int_0^1 y(\xi)d\xi\int_0^1 [Z(x,\eta)\frac{\partial K_1(x,\xi)}{\partial\xi}-$$

$$-Z(x,\eta)K_2(x,\xi)]dx=\begin{cases} y(\eta), & \text{if} \quad \eta\in(0,1), \\ \frac{1}{2}y(0), & \text{if} \quad \eta=0, \\ \frac{1}{2}y(1), & \text{if} \quad \eta=1. \end{cases} \qquad (6)$$

Indeed, to prove this statement it suffices we take into account that (4) is a fundamental solution of (3) and multiply the both hand sides of (1) by (4) and integrate the both hand sides with respect to x on $(0,1)$.

Remark 1. In the obtained expression (6) when $\eta=0$ and $\eta=1$ the expressions obtained for $\frac{1}{2}y(0)$ and $\frac{1}{2}y(1)$ are necessary conditions. We can easily see that one of these necessary conditions turns into identity, the second one takes the following form:

$$\alpha_0 y(0)+\alpha_1 y(1)=\delta-\int_0^1 p(\xi)y(\xi)d\xi, \qquad (7)$$

here

$$\alpha_0=1+\frac{1}{a(0)}\int_0^1[K_1(x,0)-b_0(x)]e^{\int_0^x\frac{b(\xi)-a'(\xi)}{a(\xi)}d\xi}dx, \qquad (8)$$

$$\alpha_1=\frac{-a(1)}{a(0)}e^{\int_0^1\frac{b(\xi)-a'(\xi)}{a(\xi)}d\xi}-\frac{1}{a(0)}\int_0^1 K_1(x,1)e^{\int_0^x\frac{b(\xi)-a'(\xi)}{a(\xi)}d\xi}dx- \qquad (9)$$

$$-\frac{1}{a(0)}\int_0^1 b_1(x)e^{\int_0^x\frac{b(\xi)-a'(\xi)}{a(\xi)}d\xi}dx,$$

$$\delta=-\frac{1}{a(0)}\int_0^1 f(x)e^{\int_0^x\frac{b(\xi)-a'(\xi)}{a(\xi)}d\xi}dx \quad , \qquad (10)$$

$$p(\xi)=\frac{1}{a(0)}\int_0^1\frac{\partial K_1(x,\xi)}{\partial\xi}e^{\int_0^x\frac{b(\xi)-a'(\xi)}{a(\xi)}d\xi}dx-\frac{1}{a(0)}\int_0^1 K_2(x,\xi)e^{\int_0^x\frac{b(\xi)-a'(\xi)}{a(\xi)}d\xi}dx. \qquad (11)$$

Investigation of boundary values and Fredholm property of the problem.

Now we consider (7), (2) as a system of linear algebraic equations and see that the following statement is true for $y(0)$ and $y(1)$

Theorem 2. *Under the conditions of theorem 1, if*

$$\Delta=\begin{vmatrix}\beta_0 & \beta_1\\ \alpha_0 & \alpha_1\end{vmatrix}\neq 0,$$

then

$$y(0)=\frac{1}{\Delta}\begin{vmatrix}\gamma & \beta_1\\ \delta & \alpha_1\end{vmatrix}-\frac{1}{\Delta}\int_0^1\begin{vmatrix}K(\xi) & \beta_1\\ p(\xi) & \alpha_1\end{vmatrix}y(\xi)d\xi,\ y(1)=\frac{1}{\Delta}\begin{vmatrix}\beta_0 & \gamma\\ \alpha_0 & \delta\end{vmatrix}-\frac{1}{\Delta}\int_0^1\begin{vmatrix}\beta_0 & K(\xi)\\ \alpha_0 & p(\xi)\end{vmatrix}y(\xi)d\xi.$$

If we put these expressions into (6) we obtain:

Theorem 3. *Under the conditions of theorem 2 the boundary value problem (1)-(2)*

$$y(\eta)=F(\eta)+\int_0^1 Q(\eta,\xi)y(\xi)d\xi, \tag{12}$$

is reduced to the Fredholm type integral equation of second kind. Thus, $F(\eta)$ *and* $Q(\eta,\xi)$ *are expressed by the fundamental solution (4) and the data of the problem (1)-(2).*

So, we found the sufficient condition for the Fredholm property of linear general boundary value problem stated for a general linear equation of first order.

Unsolved problems

Problem 1. What we can say if in the equation (1) $a(x)$ or $K_1(x,\xi)$ or both of them are not differentiable.

Problem 2. What we can say if in the equation (1) the coefficient $a(x)$ vanishes at certain points and in the set $(a,b)\subset(0,1)$.

Problem 3. Find sufficient conditions for the Fredholm property of the obtained problem when all data $a(x)$, $b(x)$, $K_1(x,\xi)$, $b_1(x)$, $f(x)$, β_0, β_1, $K(\xi)$ and γ of the boundary value problem are the same order square matrices.

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BİRİNCİ TƏRTİB ADİ XƏTTİ İNTEQRO-DİFERENSİAL VƏ SƏRHƏD QİYMƏTLİ TƏNLİK ÜÇÜN QEYRİ-LOKAL VƏ QLOBAL HƏDLƏR SAXLAYAN SƏRHƏD ŞƏRTİ DAXİLİNDƏ SƏRHƏD MƏSƏLƏSİNİN HƏLLİNİN ARAŞDIRILMASI

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XÜLASƏ

Bu məqalədə birinci tərtib ümumi, xətti, inteqro-diferensial, yüklənmiş tənlik üçün xətti ümumi qeyri-lokal və qlobal hədlər saxlayan sərhəd şərti daxilində sərhəd məsələsinə baxılmışdır.

Alınmış zəruri şərtlərin və sərhəd şərtinin köməyilə tənlikdən yüklənmiş hədlər aradan qaldırılaraq verilmiş sərhəd məsələsi ikinci növ Fredholm tipli, zəif məxsusiyyətli inteqral tənliyə gətirilir.

ИССЛЕДОВАНИЕ РЕШЕНИЯ ГРАНИЧНЫХ ЗАДАЧ ДЛЯ ИНТЕГРО-ДИФФЕРЕНЦИАЛЬНОГО НАГРУЖЕННОГО ЛИНЕЙНОГО ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ ПЕРВОГО ПОРЯДКА С ГРАНИЧНЫМИ УСЛОВИЯМИ, СОДЕРЖАЩИМИ КАК НЕЛОКАЛЬНЫЕ, ТАК И ГЛОБАЛЬНЫЕ СЛАГАЕМЫЕ

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РЕЗЮМЕ

В этой работе рассмотрена граничная задача для линейного интегро-дифференциального нагруженного уравнения первого порядка с граничными условиями, содержащими как нелокальные, так и глобальные слагаемые.

Полученные необходимые условия совместно с граничными условиями дают возможность исключить нагруженные слагаемые из уравнения и получить достаточные условия Фредгольмовости.